高精度位置天文測定のための星像解析手法の開発

Ryou Ohsawa
JASMINE project, National Astronomical Observatory of Japan

Takafumi Kamizuka
The Institute of Astronomy, The University of Tokyo

Table of Contents

Power of Astrometry

Astrometry = Strong tool to reveal the history of the Milky Way (galactic archaeology, GA)

Gaia measures proper motions and parallaxes at ~10 µas/yr & 10 µas levels.

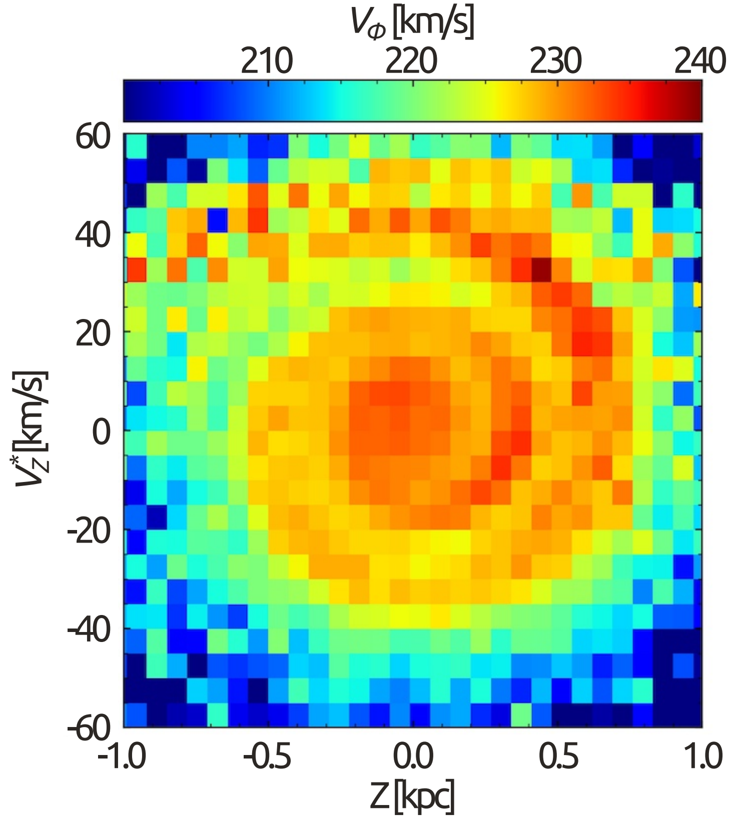
Several relics of satellite merges are discovered.

The Gaia-Sausage-Enceladus in the inner halo

The remnant of a major merger that formed the inner halo possibly occurred about 100 Gyr ago

The phase spiral of the Galactic disk

The after effect of a satellite galaxy passage about 10 Gyr ago, which disturbed the Galactic disk in a phase space



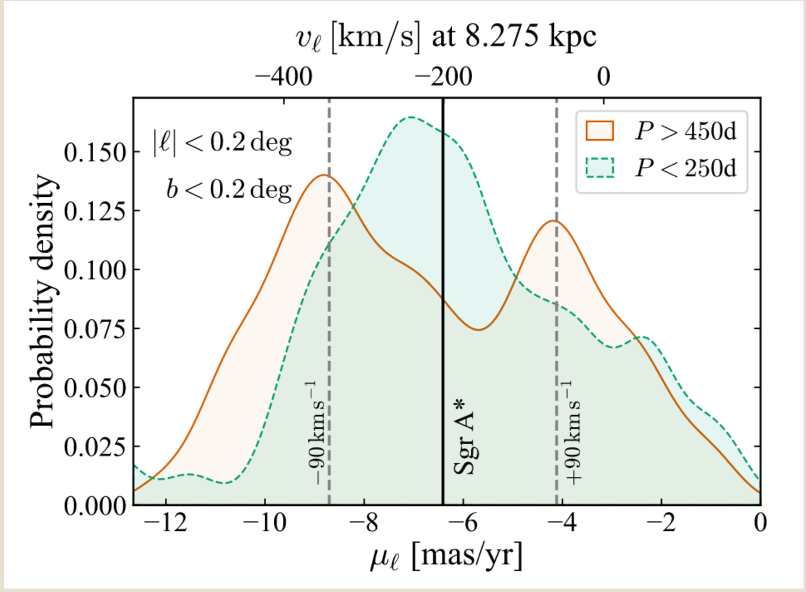
e.g., Belokurov et al. (2018), Antoja et al. (2023), Antoja et al. (2018)

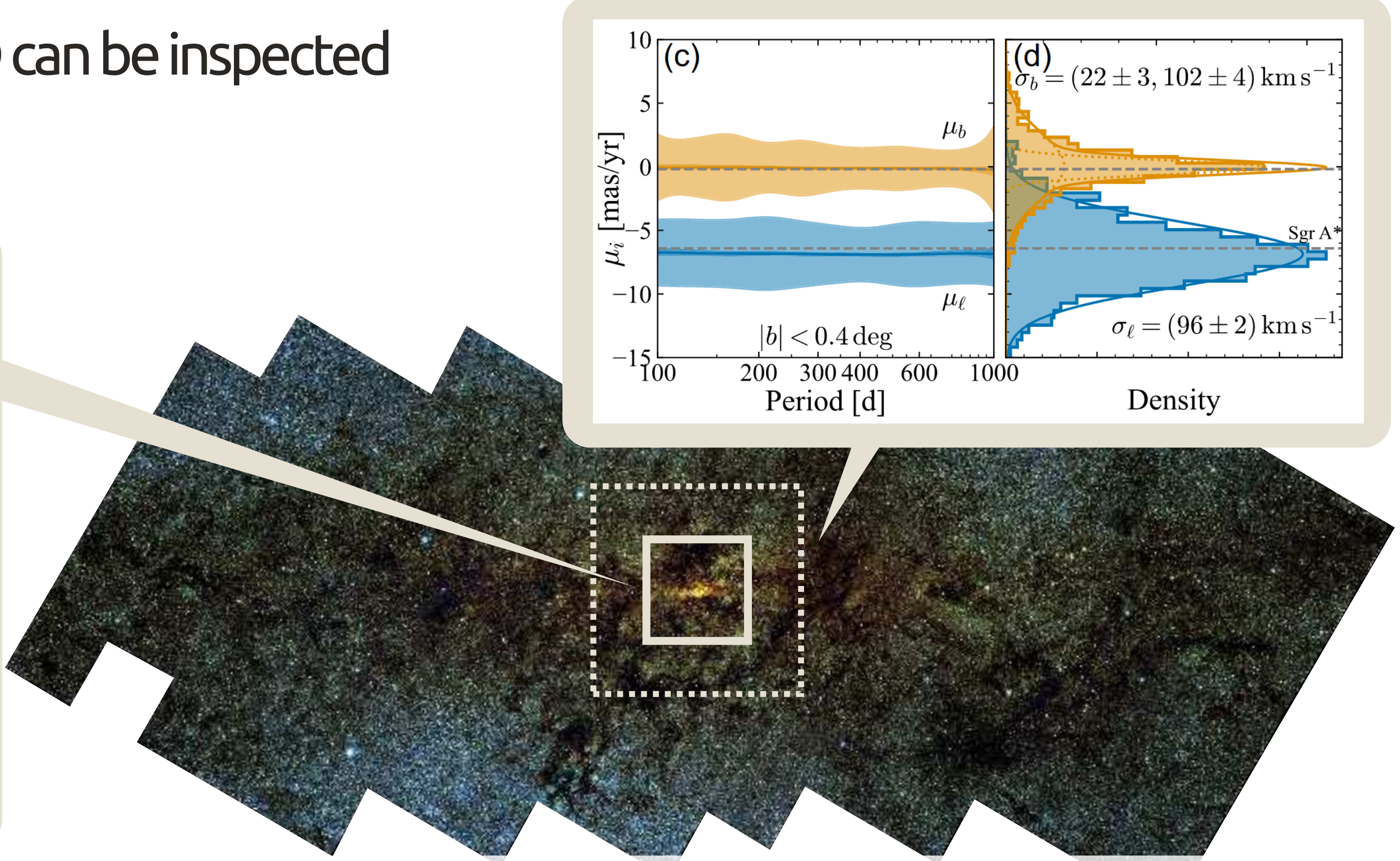
Astrometric Frontier

Galactic center regions is not inspected by *Gaia*.

Sources at > 4 kpc are heavily obscured by interstellar dust.

Formation of the bar, bulge, and NSD can be inspected by the Galactic center astrometry.





SIRIUS survey (Nishiyama+, 2006), Miras in NSD (Sanders+, 2024)

Astrometry on Images

Astronomical data = intensity distribution by image sensors.

- Photon distribution is quantized by detector pixels.
- Positions are estimated using a set of pixel values.

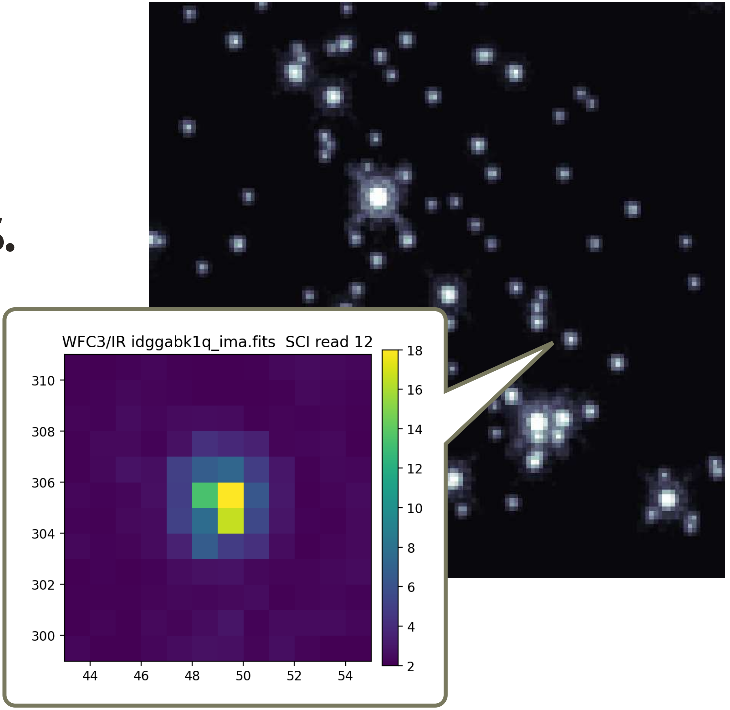
$$I_{i,j} \longmapsto (x, y)$$

$$\{\, i, j \in \mathbb{N}; \quad x, y, I_{i,j} \in \mathbb{R} \,\}$$

- An appropriate imaging model is required to reproduce the observation.

$$F(x, y, i, j, I_0) \approx I_{i,j}$$

Optimization targets



Simulated stellar field using a WFC3/IR ponit spread function model (provided by STScI)

Effective PSF

Modern astrometry requires ~0.01-pixel level measurement accuracies.

Precise point-source image modeling is required.

Point-spread function (PSF): $\varphi(x - x_0, y - y_0)$

A continuous function defined on a focal plane. x and y are forcal-plane coordinates.

The shape is defined by optics and a source spectrum.

Point-response function (PRF): $\psi(i, j, x_0, y_0)$

A function providing the photon count at (i, j) pixel. x_0 and y_0 are stellar positions.

A PRF is derived by integrating the PSF and an intraxpiel flat pattern within a (i, j) pixel.

Profile fitting with an appropriate PRF provides accurate (positional) measurements.

Effective PSF

Assuming that every pixel has the same intrapixel flat pattern, a PRF can be represented as a virtual two-dimensional image.

Effective PSF (ePSF): $\Psi(i - \delta_x, j - \delta_y)$

A two-dimensional continuous function. δ_x and δ_y are pixel phases.

$$\psi(i, j, x_0, y_0) = \int_{i-0.5}^{i+0.5} \int_{j-0.5}^{j+0.5} \eta(x-i, y-j)\phi(x-x_0, y-y_0)\, \mathrm{d}x\, \mathrm{d}y$$

$$= \Psi(i - i_0 - \delta_x, j - j_0 - \delta_y)$$

The next challenge is **estimaging the ePSF for an exposure.**

Anderson & King (2000), PASP 112, 1360

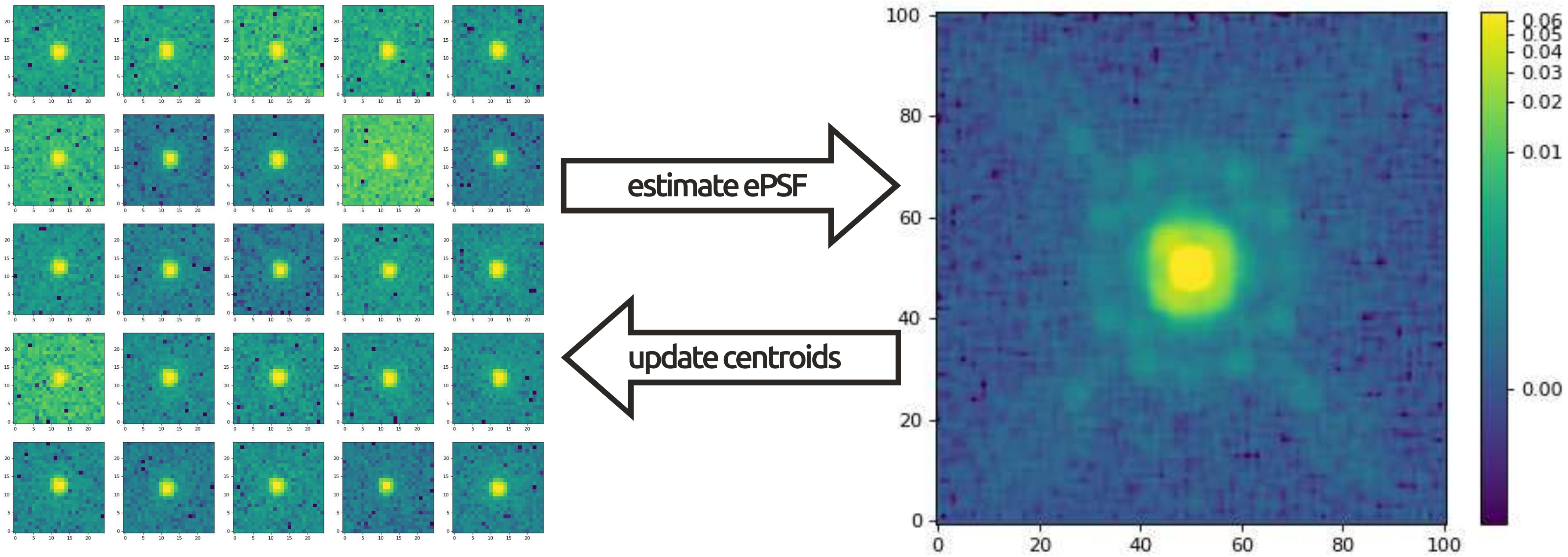
ePSF method using photutils

Anderson & King (2000, **AK20**) provided a procedure to derive ePSF in case that all the stellar images are descended from the same PRF.

The procedure is (in part) implemented in `photutils` and has been widely used.

Multiple stellar image cutouts

Estimated ePSF image



Anderson & King (2000), PASP 112, 1360; images from a tutorial in photutils docs

ePSF method using photutils

The ePSF method (`photutils`) is useful but has several drawbacks.

1. Excessive degrees of freedom
2. Pixel-phase errors
3. Identical ePSF assumption

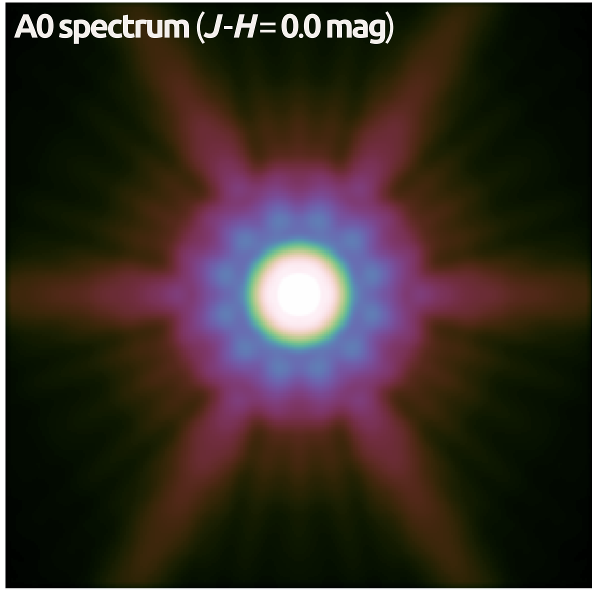
Anderson & King (2000), PASP 112, 1360

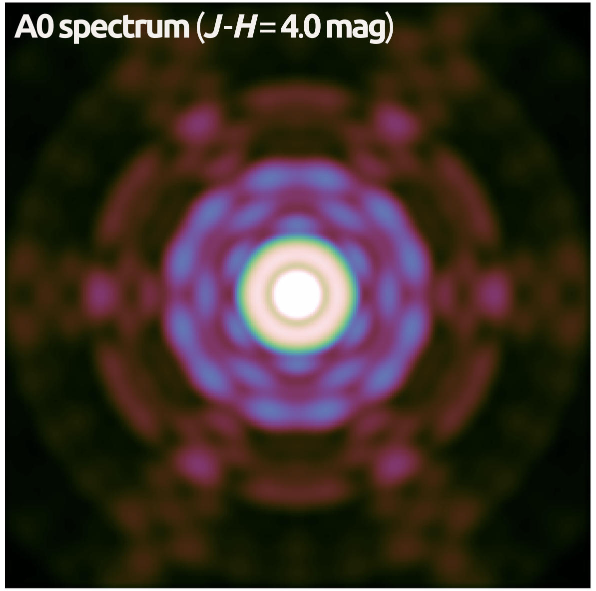
ePSF method using photutils

The ePSF method (`photutils`) is useful but has several drawbacks.

1. Excessive degrees of freedom
2. Pixel-phase errors
3. Identical ePSF assumption

ePSFs of a hypothetical telescope (JASMINE)





Anderson & King (2000), PASP 112, 1360

ePSF method using photutils

The ePSF method (`photutils`) is useful but has several drawbacks.

1. Excessive degrees of freedom
2. Pixel-phase errors
3. Identical ePSF assumption ⇨ interpolating multiple ePSFs (requireing more sources)

Purpose

Establish a method to construct a variable ePSF model from obtained images.

Anderson & King (2000), PASP 112, 1360

$\mathcal{H}$ePSF method

To construct a variable ePSF, we expand an ePSF using basis functions.

$$\Psi(x,y) \simeq \frac{1}{Z}\left[1+\sum_{2\le k+l\le M}\nu_{k,l}\,\mathcal{H}_k\left(\frac{x}{\sigma_x}\right)\mathcal{H}_l\left(\frac{y}{\sigma_y}\right)\right]\exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2}\right)\right]$$

- $\mathcal{H}_k$ is the k-th order of Hermite polynomical function.
- The shape parameters are σ_x, σ_y, and $\nu_{k,l}$, and Z is the normalizing factor.
- $k+l=1$ terms are intentionally omitted since they easily translate the centroids.
- Color depdenence was implemented as follows:

$$\sigma_x(c)=\sigma_x^{(0)}+\beta_x c,\quad \sigma_y(c)=\sigma_y^{(0)}+\beta_y c,\quad \nu_{k,l}(c)=\nu_{k,l}^{(0)}+\beta_{k,l}c$$

Refregier (2003) used a similar method to model galaxy shapes.
With a sufficient number of basis functions, this expression can approximate complicated ePSFs.

Refregier (2003), MNRAS 338, 35

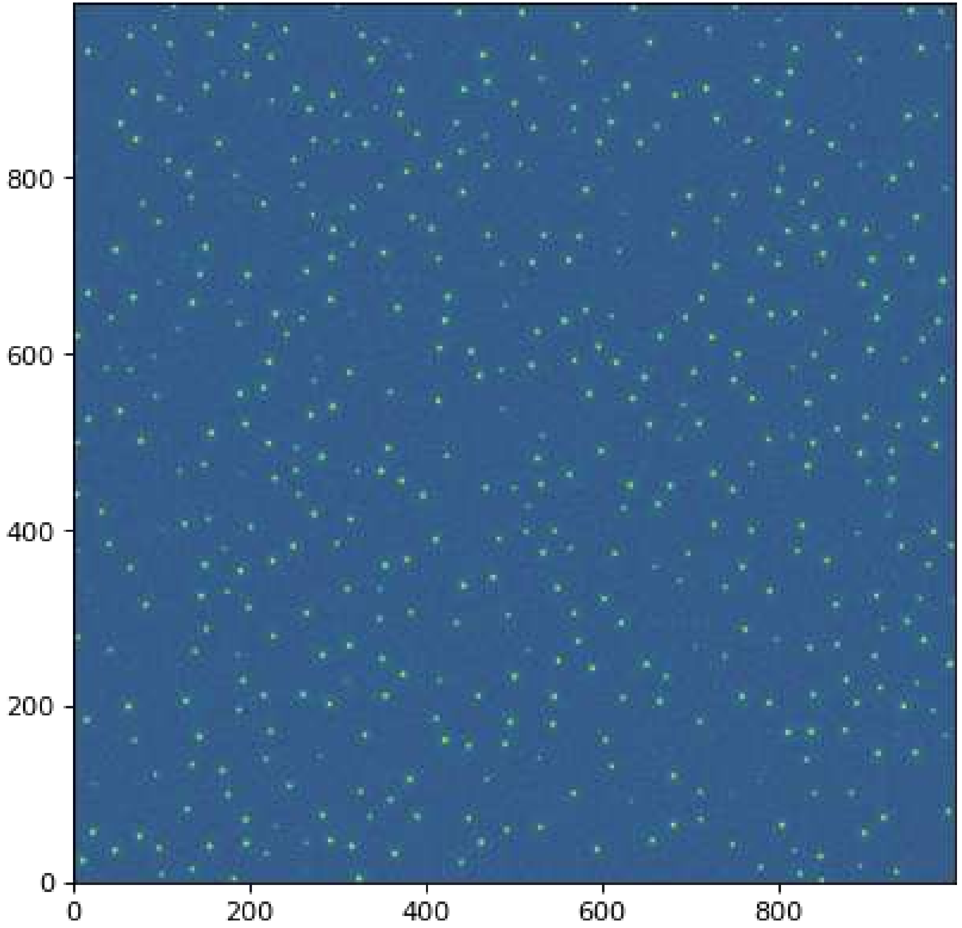
$\mathcal{H}$ePSF method

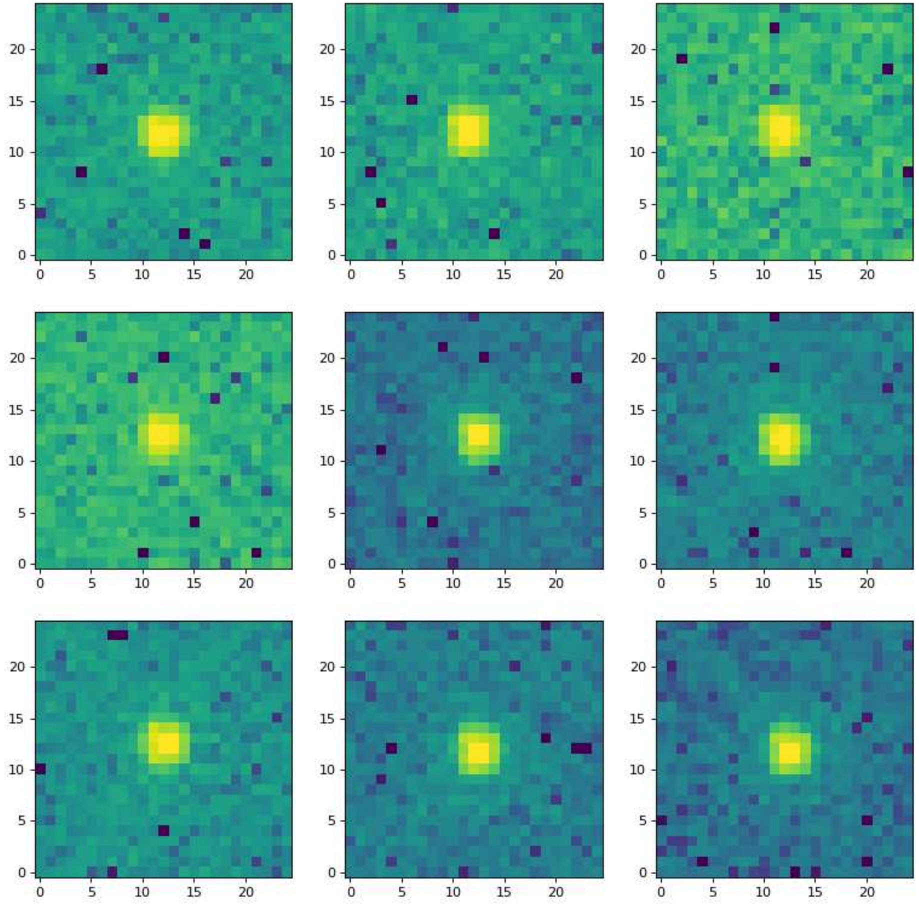
Characteristics of two approaches are compared below:

	ePSF (photutils)	$\mathcal{H}$ePSF
ePSF implementation	non-parametric	parametric
Degrees of freedom	large (> a few hundred) $\propto$ image size^2	handful (~ 10–200)
Model optimization	alternatively optimized using an efficient iterative procedure	simultaneously optimized using gradient-based methods
variable ePSF modeling	interpolating multiple ePSFs large sample required	easily implemented by adding additional paraemters
availability	implemented in photutils	not available yet

Verification with a static case

First, a HST mock image was used for performance validation.

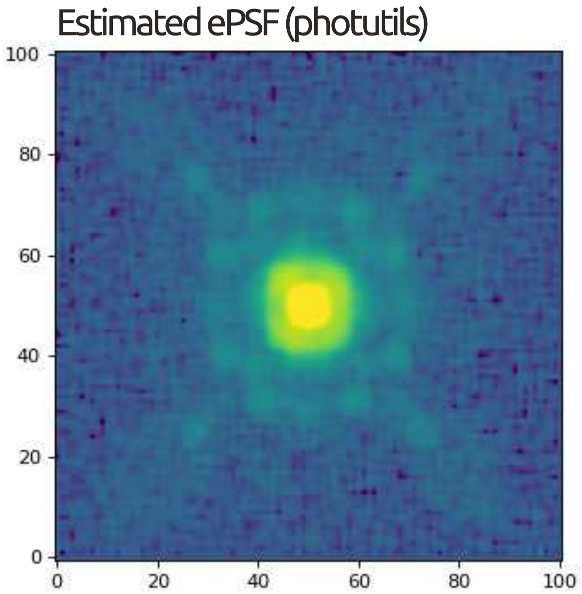




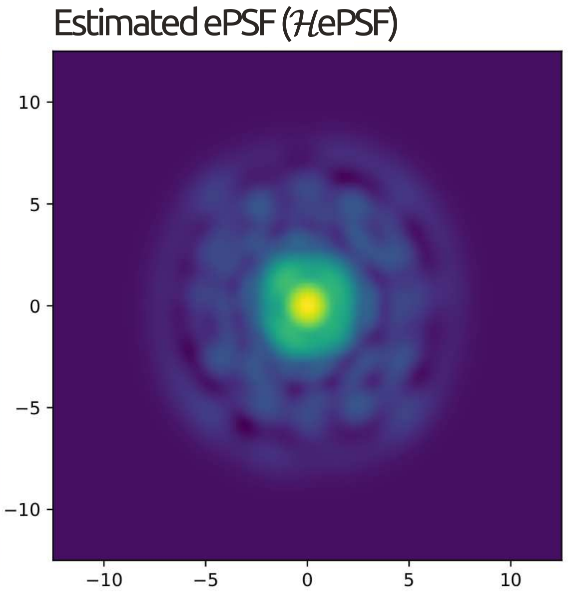
HST mock image provided by Photutils (Bradley et al., 2025)

Verification with a static case

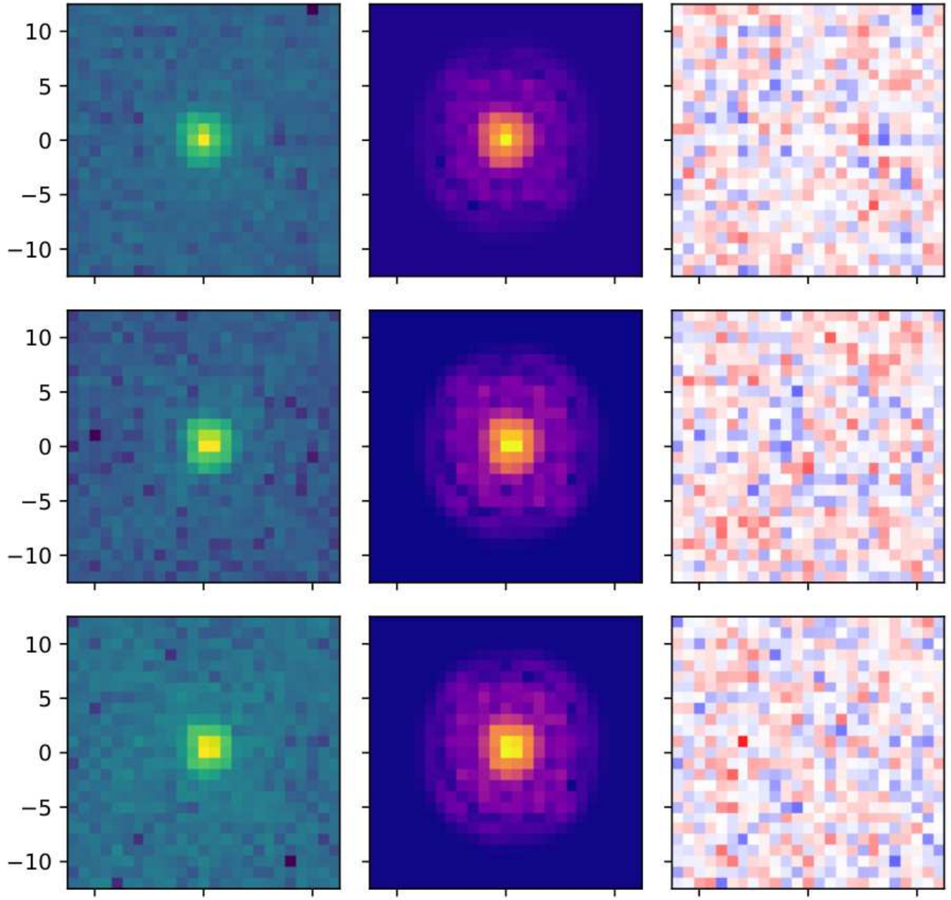
A complicated ePSF shape was successfully reproduced with a smaller d.o.f.



25×25 pixel (×4 oversampling)
degrees of freedom ≈ 10,000
3 iterations using 403 stars
log-scale (99.0 %)



covering 21×21 pixel area
degrees of freedom ≈ 228
optimized using 50 stars
log-scale (99.0 %)



ePSF from a tutorial in photutils docs

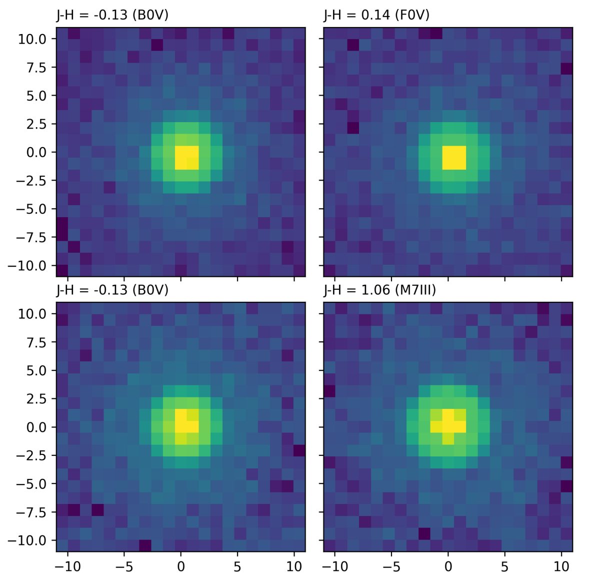
Verification with realistic cases

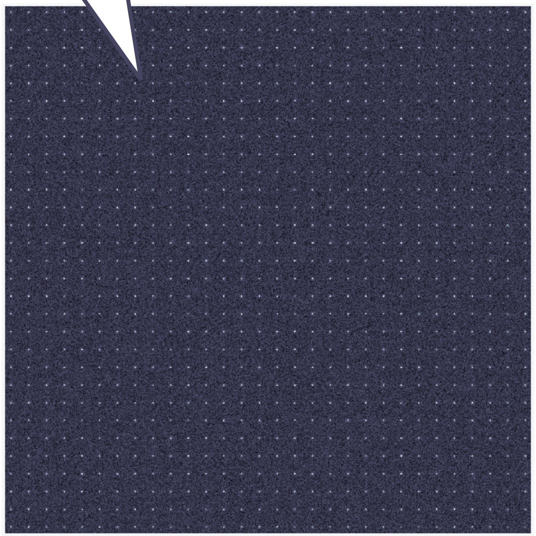
Images of stars with different spectral types were simulated.

Infrared (J-H) colors ranged from -0.12 to 0.95.

$\mathcal{H}$ePSF model was constructed using 2–16 order terms.

d.o.f = 304 including color dependence





Simulation parameters

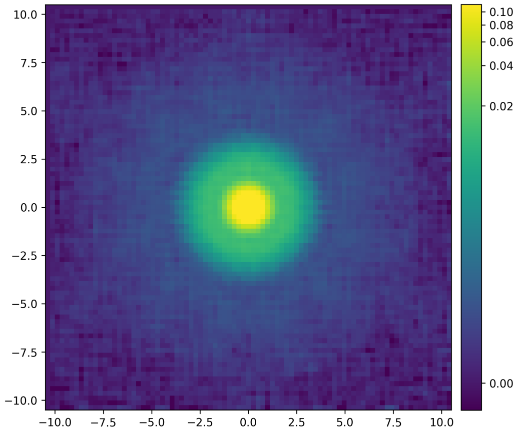
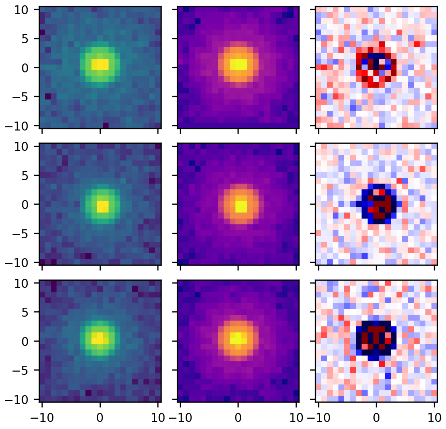
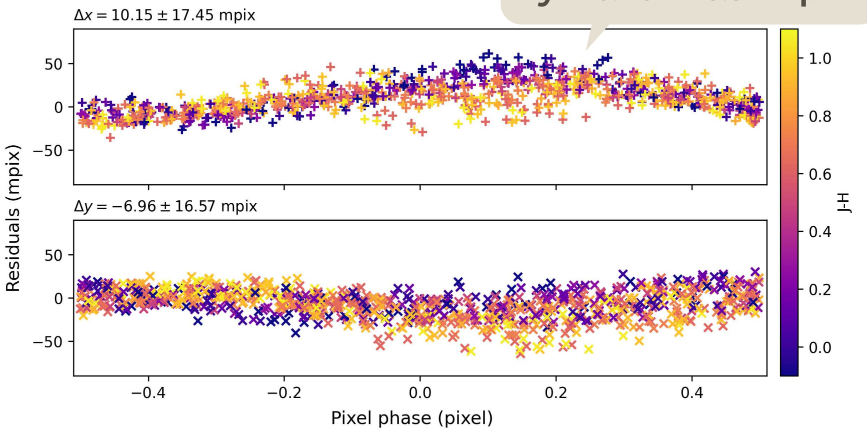
Number of sources	900
Image dimension	21 × 21 pixels
Stellar spectrum	B0V, A0V, ..., M9III (J-H = -0.12, ..., 0.95)
Pixel phases (δ_x, δ_y)	sampled from (-0.5, 0.5) uniform distribution
Source flux	sampled from $\mathcal{N}(10^5, 2{\times}10^4)$
Background noise	sampled from $\mathcal{N}(0.0, 10.0)$
Flat-fielding error	sampled from $\mathcal{N}(0.0, 0.001)$

Verification with realistic cases

Reconstructed ePSF (*J*-*H*=0) | Pixel-by-pixel residuals | Positional measurement errors

photutils

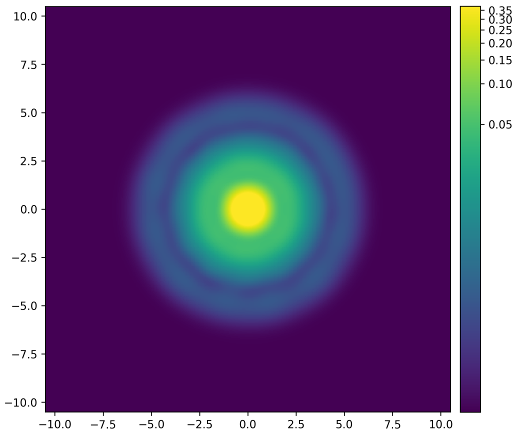
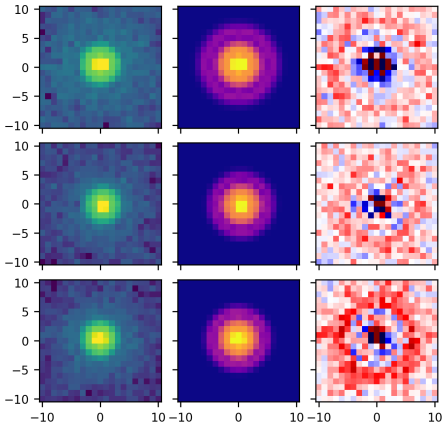
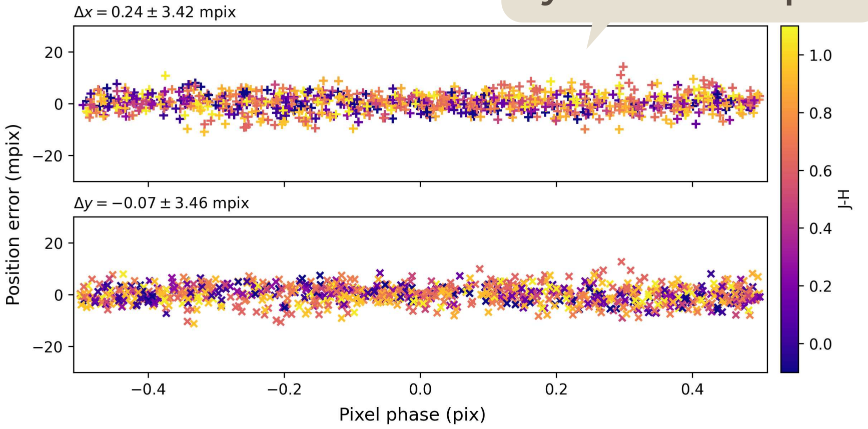
- 21×21 pixels (×4 oversampling)
- degree of freedom = 7225
- reconstructed using 100 sources
- no color dependence
- updated 10 iterations
- fitting aperture radius = 7 pix

Δx = 10.15 ± 17.45 mpix
Δy = -6.96 ± 16.57 mpix

*H*ePSF

- covering 21×21 pixels
- degree of freedom = 304
- reconstructed using 100 sources
- simple J-H color dependence
- optimized using Adam

Δx = 0.24 ± 3.42 mpix
Δy = -0.07 ± 3.46 mpix

Discussion

$\mathcal{H}$ePSF successfully reconstructed complicated, variable ePSFs.

- high astrometric accuracies with relatively small degrees of freedom
- adjustable model flexibility
- Hermite polynomials $\Rrightarrow$ inside-out efficient reconstruction
- expandability for additional parameters (e.g., flux)

Limitations of the $\mathcal{H}$ePSF method need to be investigated.

- highly extinguished cases like stars in the Galactic center region.
- possible degeneracy due to excessive degrees of freedom
- robust optimization scheme against outliers
- performance verification with telescope jitters

Summary

We propose an approach to model a variable ePSF.

- An ePSF is expanded using Hermite polynomials.
- Expansion coefficients are defined as functions with additional parameters.

$$\Psi(x,y) \simeq \frac{1}{Z}\left[1 + \sum_{2\leq k+l\leq M} \nu_{k,l}\,\mathcal{H}_k\left(\frac{x}{\sigma_x}\right)\mathcal{H}_l\left(\frac{y}{\sigma_y}\right)\right]\exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2}\right)\right]$$

The $\mathcal{H}$ePSF model successfully worked with variable ePSF cases.

- The color dependence of the ePSF was successfully reproduced.
- The $\mathcal{H}$ePSF model achieved better positional measurements than a conventional method.
- We provided a new approach to model the point-response function.

This research is in part supported by NAOJ Japanese TMT Office Strategic Research and Development Funding of FY2025
Numerical computations were [in part] carried out on GPU cluster at the CfCA/NAOJ and MDAS operated by the ADC/NAOJ

$\mathcal{H}$ePSF method

To construct a variable ePSF, we expand an ePSF using basis functions.

$$\Psi(x,y) \simeq \frac{1}{Z}\left[1+\sum_{2\le k+l\le M}\nu_{k,l}\,\mathcal{H}_k\!\left(\frac{x}{\sigma_x}\right)\mathcal{H}_l\!\left(\frac{y}{\sigma_y}\right)\right]\exp\!\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2}\right)\right]$$

$\mathcal{H}_k$ is the k-th order of Hermite polynomical function.

Refregier (2003) used a similar method to model galaxy shapes. Including a sufficient number of basis functions, this expression can approximate complicated ePSF.

The expansion coefficiets $\nu_{k,l}$ can be a function of additional parameters (e.g., colors).

Refregier (2003), MNRAS 338, 35

$\mathcal{H}$ePSF method

$$\Psi(x,y) \simeq \frac{1}{Z}\left[1+\sum_{2\leq k+l\leq M}\nu_{k,l}\,\mathcal{H}_k\left(\frac{x}{\sigma_x}\right)\mathcal{H}_l\left(\frac{y}{\sigma_y}\right)\right]\exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2}\right)\right]$$

- The shape parameters are σ_x, σ_y, and $\nu_{k,l}$, and Z is the normalizing factor.
- $k+l=1$ terms are intentionally omitted since they easily translate the centroids.
- Color depdenence was implemented as follows:

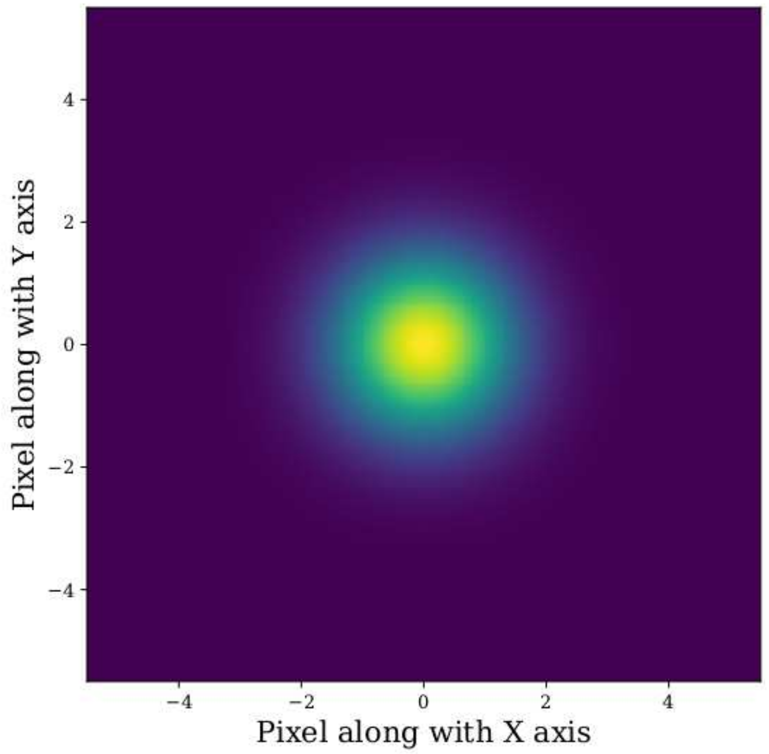
$$\sigma_x(c)=\sigma_x^{(0)}+\beta_x\,c,\quad \sigma_y(c)=\sigma_y^{(0)}+\beta_y\,c,\quad \nu_{k,l}(c)=\nu_{k,l}^{(0)}+\beta_{k,l}\,c$$

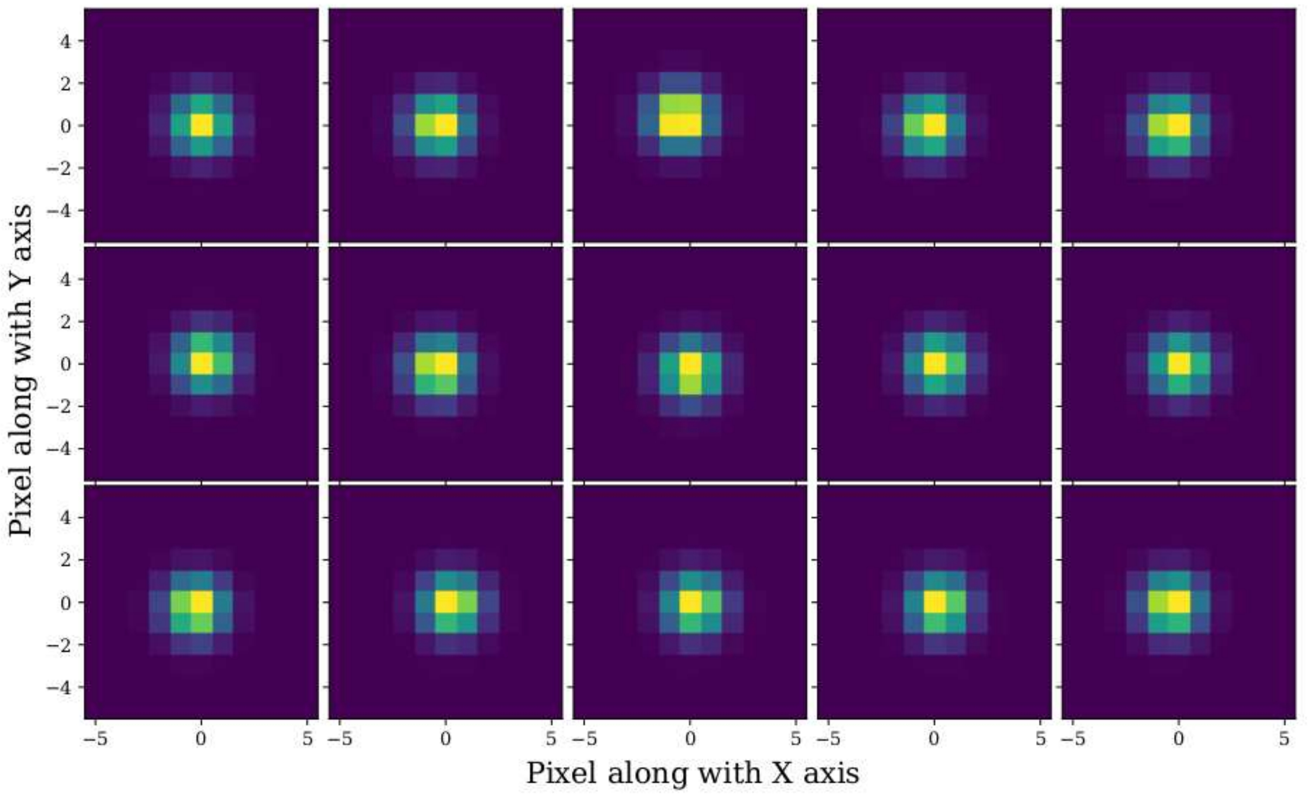
- The parameters were optimized by minimizing the squared sum of residuals.
- The Levenberg-Marquardt algorithm was used.

Gaussian PRF case

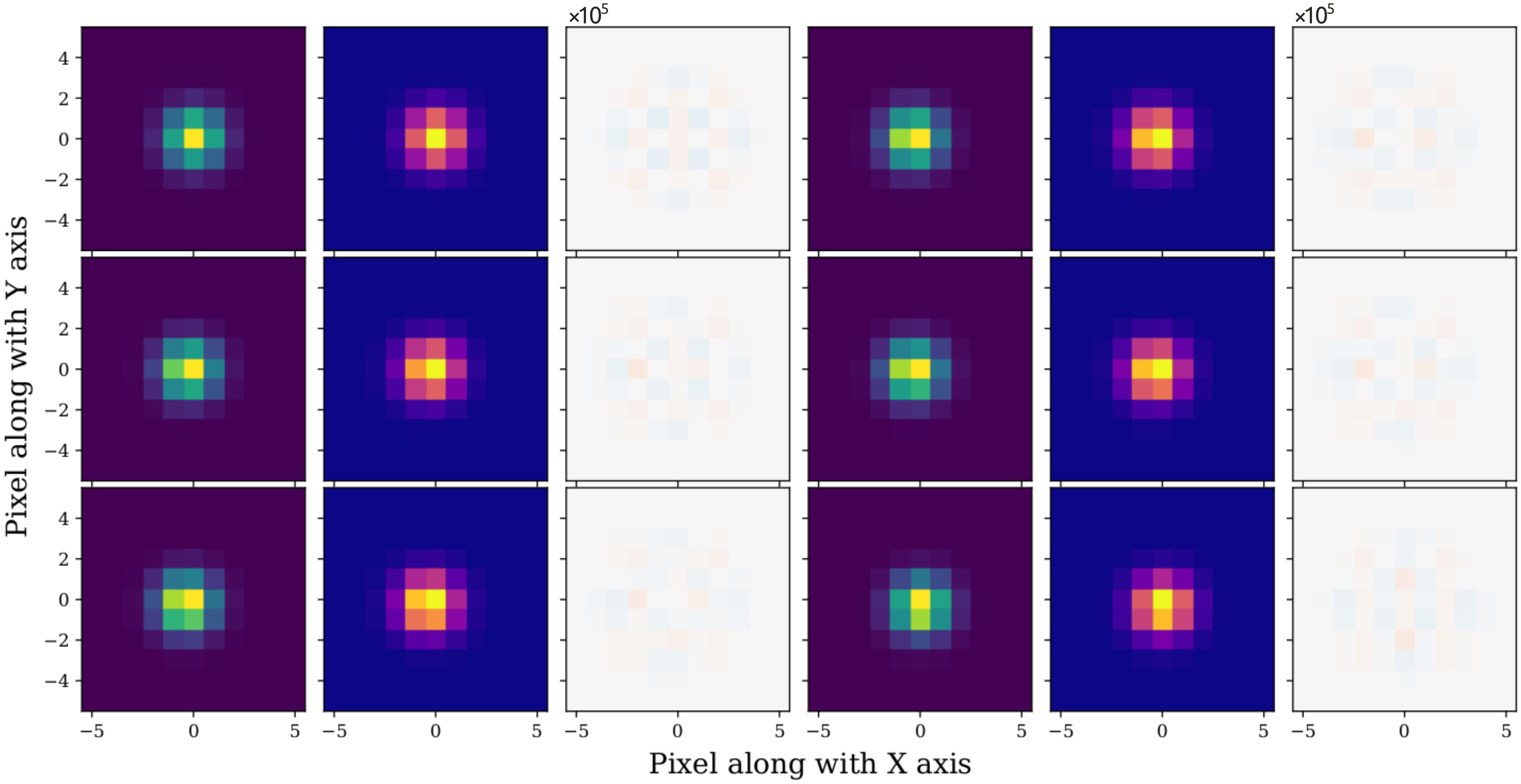
Simulated images with a 2D Gaussian PRF:

$$\psi(i,j,\delta_x,\delta_y) = \frac{1}{4}\left[\mathrm{erf}\left(\frac{i-\delta_x+0.5}{\sqrt{2}\sigma_x}\right) - \mathrm{erf}\left(\frac{i-\delta_x-0.5}{\sqrt{2}\sigma_x}\right)\right]\left[\mathrm{erf}\left(\frac{j-\delta_y+0.5}{\sqrt{2}\sigma_y}\right) - \mathrm{erf}\left(\frac{j-\delta_y-0.5}{\sqrt{2}\sigma_y}\right)\right]$$

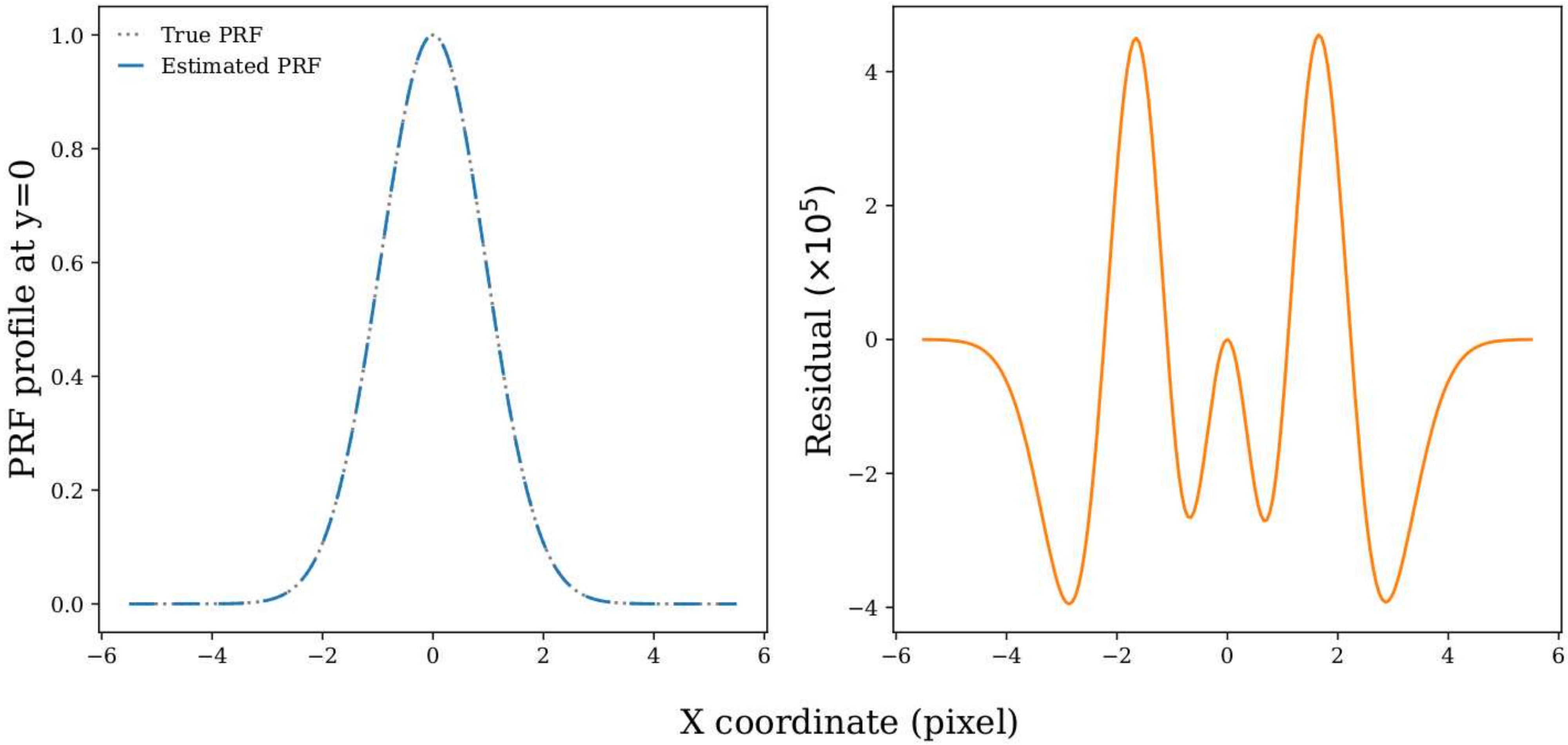




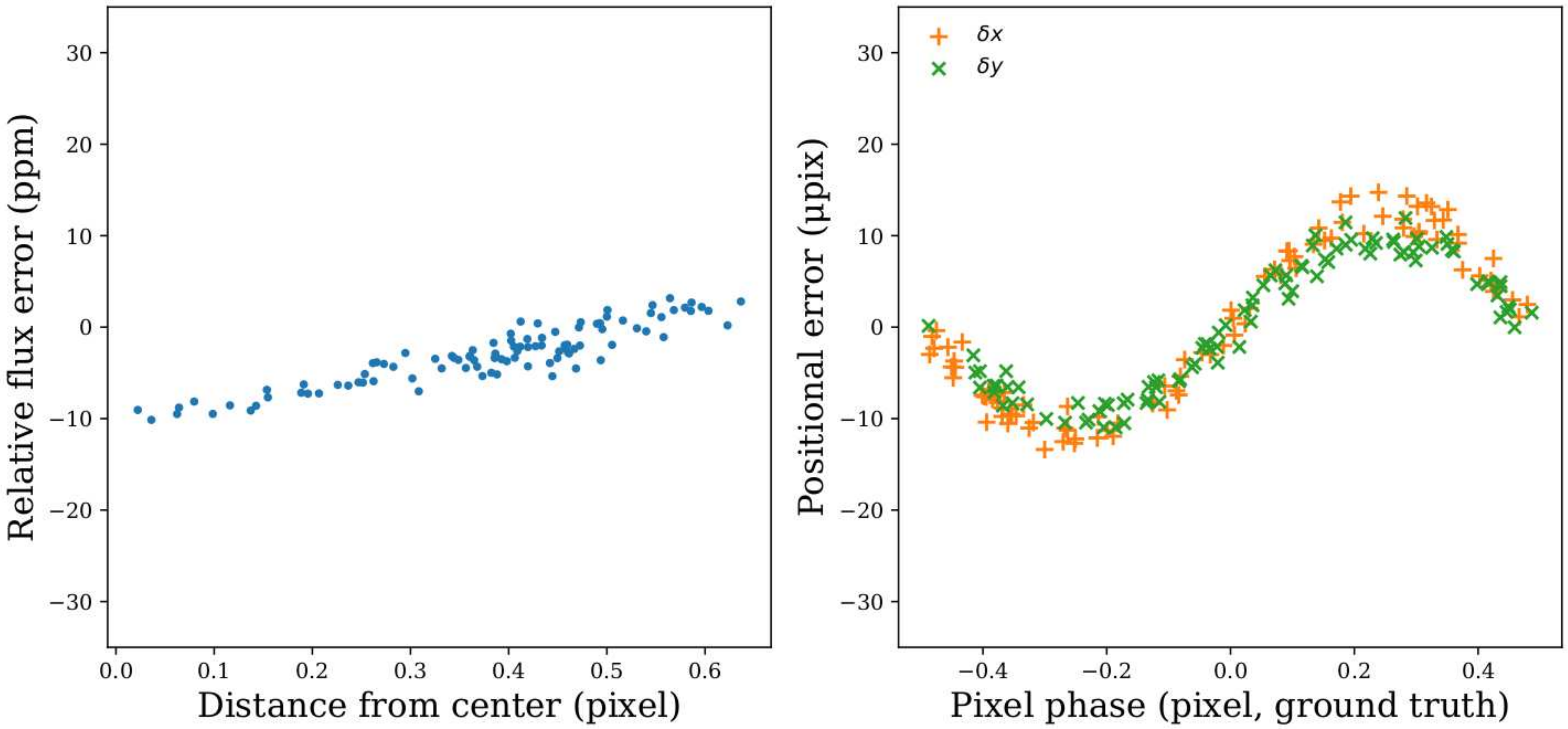
Gaussian PRF case



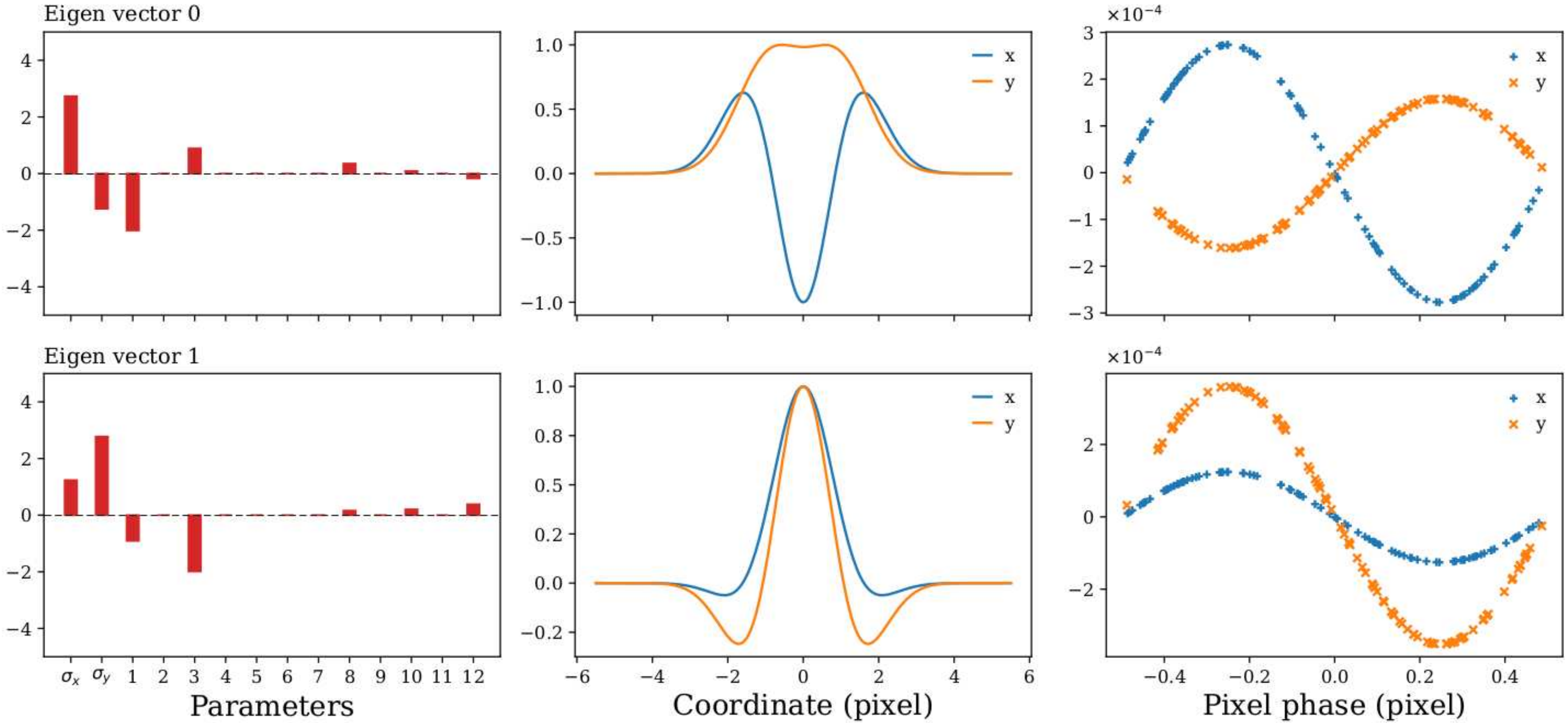
Gaussian PRF case



Gaussian PRF case



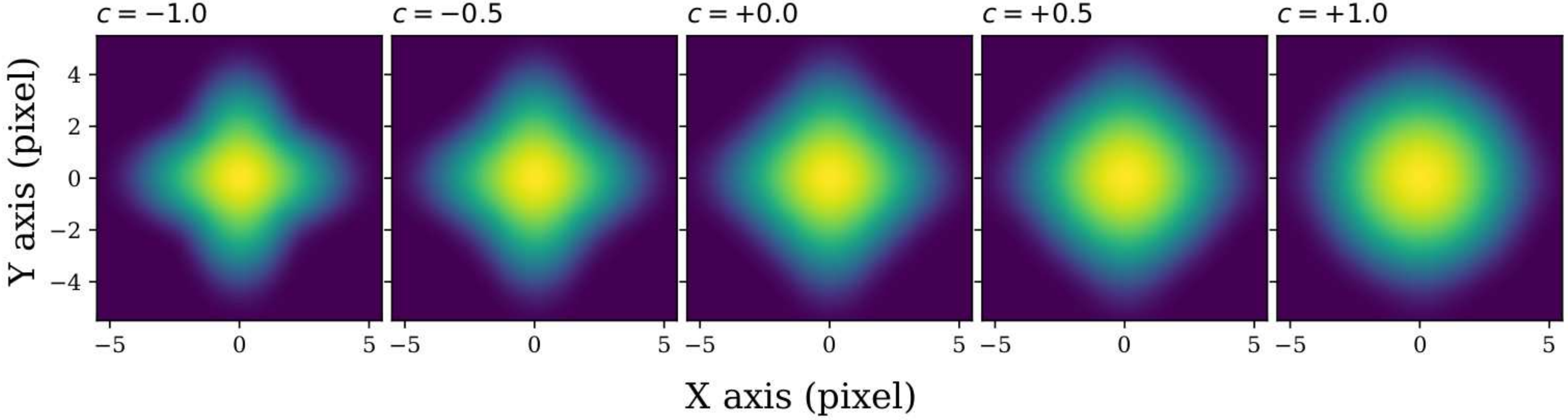
Gaussian PRF case



Verification with a variable PRF

As a concept verification, we applied *H*ePSF method to simulated data.

An artificial ePSF with a color parameter was used to generate stellar images.



Color parameters were sampled from a uniform distribution (-1, 1).

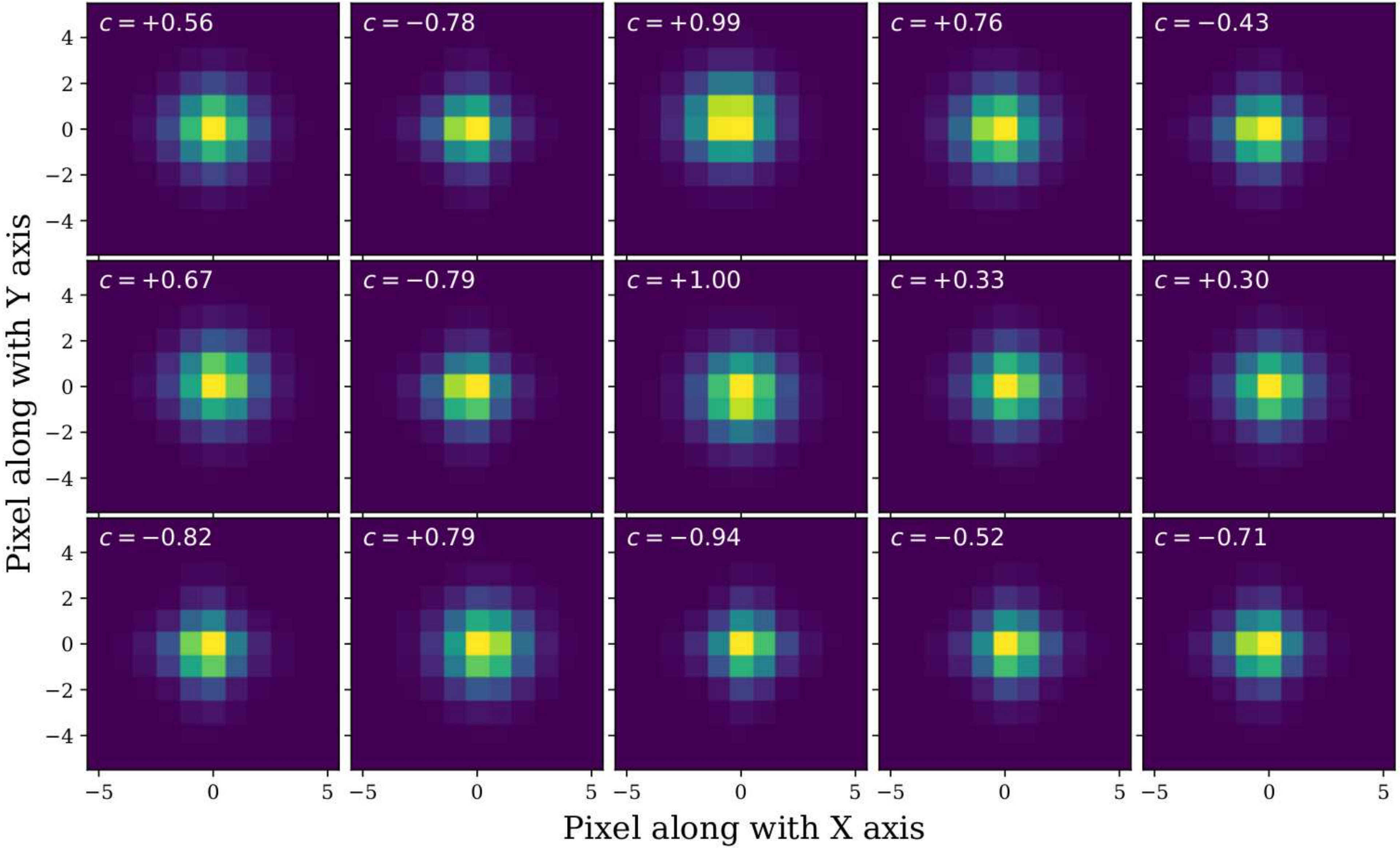
100 stellar images were generated and their positions were measured.

Variable PRF case: simulation

Details of simulated data are listed in the table below.
For simplicity, solely photon and background noise is considered.

	Specifications
Number of sources	100
Image dimension	11 × 11 pixels
ePSF model	variable, symmetric along x- and y-axes
Color parameter	sampled from $(-1, 1)$ uniform distribution
Pixel phases (δ_x, δ_y)	sampled from $(-0.5, 0.5)$ uniform distribution
Source intensity	sampled from $\mathcal{N}(10^6, 10^4)$
Background noise	sampled from $\mathcal{N}(0.0, 10.0)$

Exapmles of generated images



Variable PRF case: model setup

*H*ePSF model with $M\!=\!4$ was applied, where 12 basis functions were included.

Considering the xy symmetry, only the even terms were set color-dependent.

The *H*ePSF model contained 22 shape parameters.

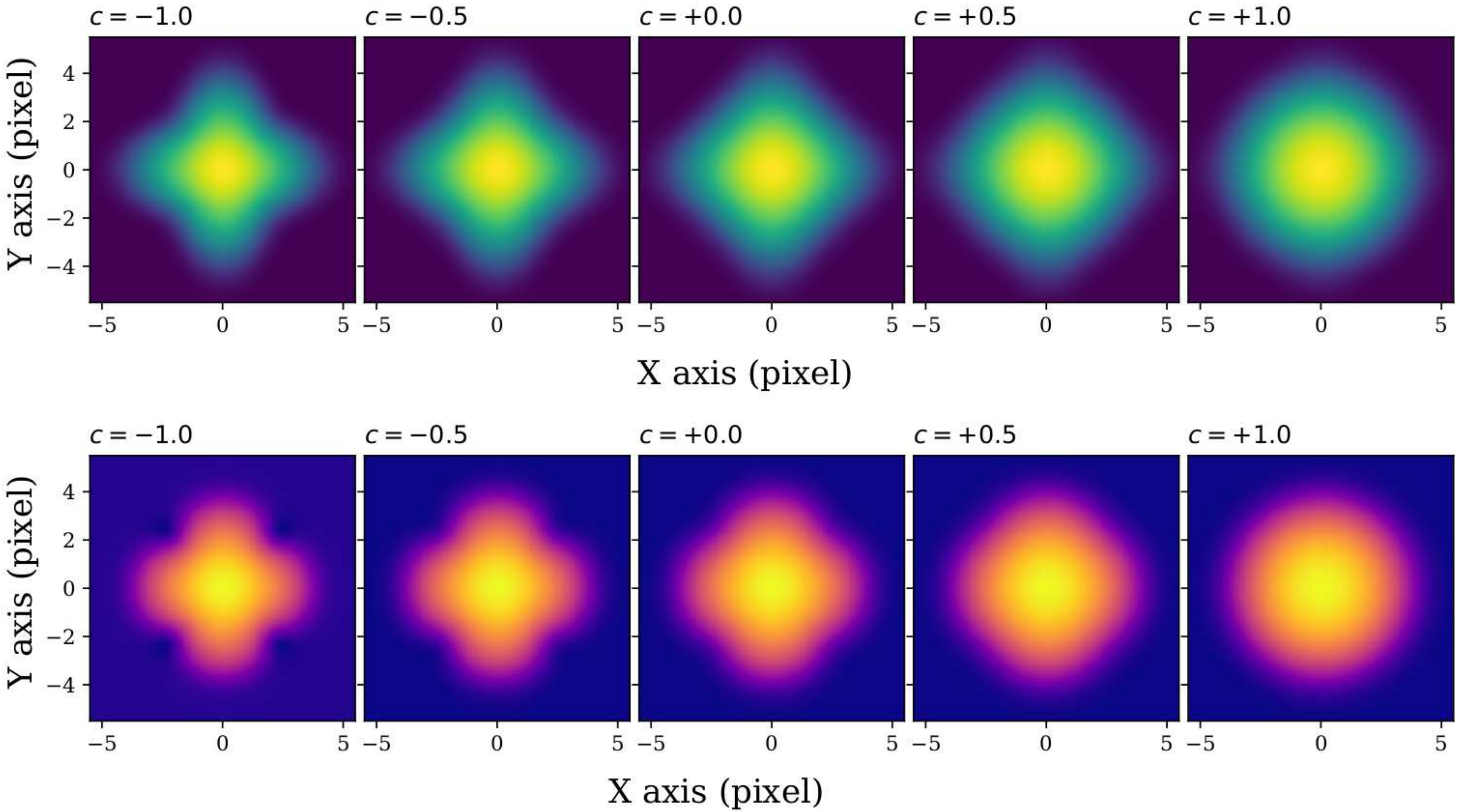
2 scale parameters (σ_x, σ_y), 12 color-independent coefficients, 8 color-dependent coefficients.

The model parameters were simultaneously optimized with source intensities and positions using the least-squares method. The colors were treated as known parameters.

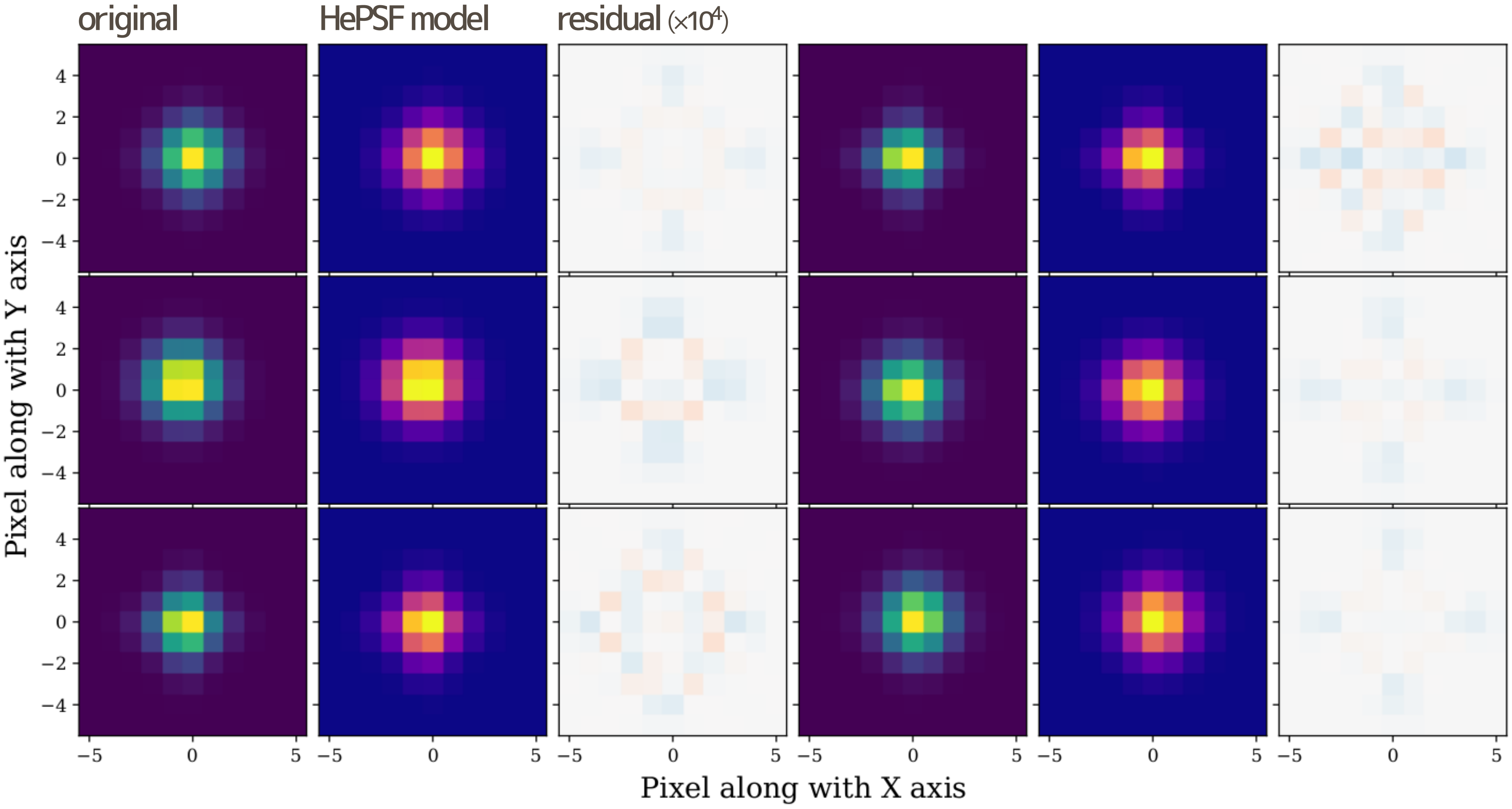
$M\!=\!2$ terms	$x^2-1,\ xy,\ y^2-1$
$M\!=\!3$ terms	$x^3-3x,\ x^2y-y,\ xy^2-x,\ y^3-3y$
$M\!=\!4$ terms	$x^4-6x^2+3,\ x^3y-3xy,$ $x^2y^2-x^2-y^2+1,\ xy^3-3xy, y^4-6y+3$

Variable PRF case: reconstruction

Original (top) and reconstructed (bottom) ePSF



Variable PRF case: residuals



Variable PRF case: deviations